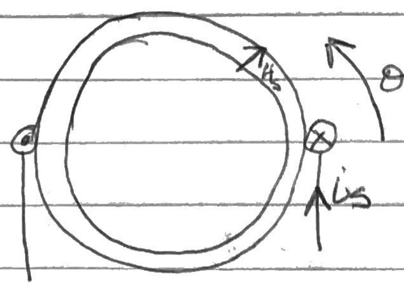
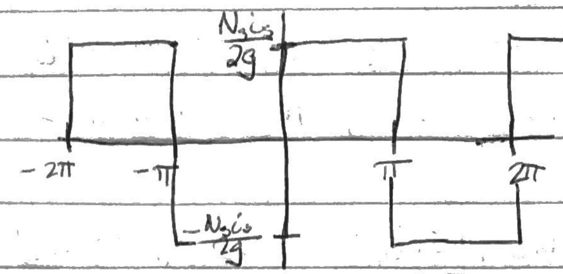
Single Phase Stator Magnetic Field





$$H_s \approx \frac{N_s i_s}{2g} \sin(\theta)$$

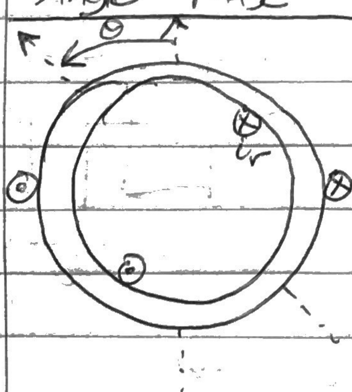
* Assume $i_s = I_s \cos(\omega_s t)$

$$H_s = \frac{N_s I_s}{2g} \cos(\omega t) \sin(\theta)$$

at time $t=0$, H_s is max at $\theta = \frac{\pi}{2}$

at time $t=\varepsilon$, H_s is max at $\theta = \frac{\pi}{2}$

Single Phase rotor-Stator:



$$\lambda_s = L_s i_s + M\cos(\theta) i_r$$

$$\lambda_r = M\cos(\theta) i_s + L_r i_r$$

$$W_m' = \frac{1}{2}(L_s i_s^2 + L_r i_r^2) + M\cos(\theta) i_s i_r$$

$$T^e = -M\sin(\theta) i_s i_r$$

Assume: $i_s = I_s \cos(\omega_s t)$ $\quad i_r = I_r \cos(\omega_r t)$

$$T^e = -M I_s I_r \cos(\omega_s t)\cos(\omega_r t)\sin(\theta)$$

Power: $P = T^e \frac{d\theta}{dt} \Rightarrow P = T^e \omega_m$

Assume: $\theta = \omega_m t + \gamma$

$P = -\omega_m M I_s I_r \cos(\omega_s t)\cos(\omega_r t)\sin(\omega_m t + \gamma)$

* Simplifying using trig identities gives

$$\boxed{P = -\tfrac{1}{4}\omega_m M I_s I_r \Big[\sin((\omega_s + \omega_r + \omega_m)t + \gamma) + \sin((\omega_s - \omega_r + \omega_m)t + \gamma) + \sin((\omega_m - \omega_s - \omega_r)t + \gamma) + \sin((-\omega_s + \omega_r + \omega_m)t + \gamma)\Big]}$$

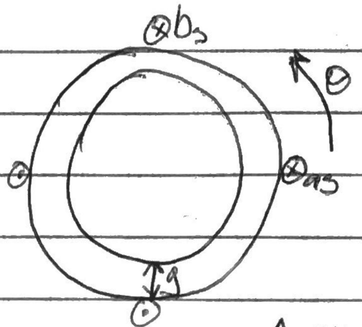
* For a non-zero average power, 1 of the combinations of $\omega_s, \omega_m, \omega_r$ needs to be zero

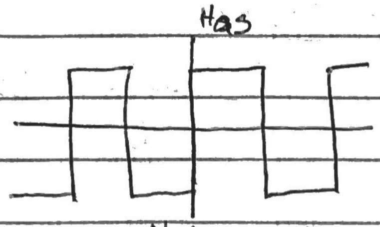
$P_{avg} = -\tfrac{1}{4}\omega_m M I_s I_r \sin(\gamma)$.

* But, will have pulsating Power (and Torque) due to the other combinations of $\omega_s, \omega_m, \omega_r$.
* Bad for the shaft. How to fix this?

2 Phase Stator

[redacted] Magnetic field:





$H_{as} \approx \frac{N_s i_{as}}{2g}\sin(\theta)$ $\qquad$ $H_{bs} \approx \frac{N_s i_{bs}}{2g}\cos(\theta)$

$H_s = H_{as} + H_{bs}$

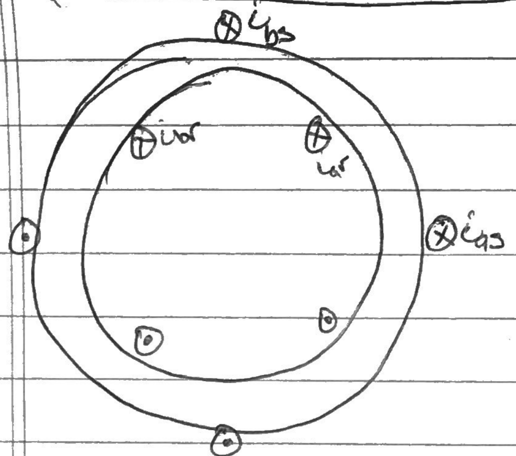
*Assume: $i_{bs} = I_s\cos(\omega_s t)$ $\qquad$ $i_{as} = I_s \sin(\omega_s t)$

$$H_s = \frac{N_s I_s}{2g}\left[\sin(\theta)\sin(\omega_s t) + \cos(\theta)\cos(\omega_s t)\right]$$

$$H_s = \frac{N_s I_s}{2g}\cos(\omega_s t - \theta)$$

at $t=0$, max H_s at $\theta = 0$; at $t = \varepsilon$, max H_s at $\theta = \omega_s \varepsilon$.

Two Phase Rotor–Stator:



$$\begin{bmatrix} \lambda_{as} \\ \lambda_{br} \\ \lambda_{ar} \\ \lambda_{br} \end{bmatrix} = \begin{bmatrix} L_s & 0 & M\cos(\theta) & -M\sin(\theta) \\ 0 & L_s & M\sin(\theta) & M\cos(\theta) \\ M\cos(\theta) & M\sin(\theta) & L_r & 0 \\ -M\sin(\theta) & M\cos(\theta) & 0 & L_r \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{ar} \\ i_{br} \end{bmatrix}$$

$$W_m' = \frac{1}{2}\left[L_s(i_{as}^2 + i_{bs}^2) + L_r(i_{ar}^2 + i_{br}^2)\right] + M\cos(\theta)\, i_{as} i_{ar} + M\sin(\theta)\, i_{bs} i_{ar}$$
$$- M\sin(\theta)\, i_{as} i_{br} + M\cos(\theta)\, i_{bs} i_{br}$$

$$T^e = \frac{\partial W_m'}{\partial \theta} = -M\sin(\theta)\left[i_{as} i_{ar} + i_{bs} i_{br}\right] - M\cos(\theta)\left[i_{as} i_{br} - i_{bs} i_{ar}\right]$$

<u>Assume:</u> $i_{as} = I_s \cos(\omega_s t)$ $\quad$ $i_{ar} = I_r \cos(\omega_r t)$

$i_{bs} = I_s \sin(\omega_s t)$ $\quad$ $i_{br} = I_r \sin(\omega_r t)$

$\theta = \omega_m t + \gamma$ $\qquad$ $\frac{d\theta}{dt} = \omega_m$

$$\Rightarrow T^e = -M I_s I_r \sin\left((\omega_m - \omega_s + \omega_r)t + \gamma\right)$$

<u>Frequency condition:</u> $\omega_m = \omega_s - \omega_r$

$T^e = -M I_s I_r \sin(\gamma)$ $\qquad$ constant torque

$P = -\omega_m M I_s I_r \sin(\gamma)$ $\qquad$ average power